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PRESSURE EXCURSIONS IN TRANSIENT FILM BOILING FROM A SPHERE TO A SUBCOOLED LIQUID

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Abstract—The maximum pressure excursions which occur in the vapor film enveloping a very hot solid sphere of nearly constant temperature which is suddenly exposed to a pool of stagnant and subcooled liquid are analytically evaluated along with related quantities. Generalized predictions for the film pressure's frequency of oscillation and maximum excursion are presented in graphical and approximate equation forms. The sphere surface temperature drop is estimated in an approximate manner and is shown to become appreciable when the liquid's subcooling exceeds a threshold value.

NOMENCLATURE

A,	parameter, $A^2 = P_\infty/R\rho\delta_0$;
B,	parameter, $B = (T_s - T_\infty)/\ln(P_v/P_{vo})$;
C,	liquid specific heat;
C_s,	sphere specific heat;
e,	constant, $e = 2.7182\ldots$;
F,	dimensionless parameter, $F = \alpha/[1+\beta(4\alpha/3)^{1/2}\mathrm{d}I/\mathrm{d}W\vert_{\mathrm{ave}}]$;
I,	square root of dimensionless integral;
i,	dimensionless integral;
k,	liquid thermal conductivity;
k_s,	sphere thermal conductivity;
k_v,	vapor thermal conductivity;
M,	vapor mass per unit area;
M_g,	noncondensable gas mass per unit area;
M_0,	initial vapor mass per unit area;
m,	dimensionless vapor mass per unit area, $m = M/\rho_{v0}\delta_0$;
P,	total film pressure;
P_g,	partial pressure of noncondensable gas;
P_{g0},	initial partial pressure of noncondensable gas;
P_v,	vapor pressure;
P_{v0},	initial vapor pressure;
P_∞,	pressure far from sphere;
p,	dimensionless total film pressure, $p = P/P_\infty$;
p_m,	maximum value of p;
p_{v0},	dimensionless initial vapor pressure, $p_{v0} = P_{v0}/P_\infty$;
q_v,	heat flow rate into interface from vapor;
R,	sphere radius;
R_g,	noncondensable gas's gas constant;
R_v,	vapor's gas constant;
T,	absolute temperature;
T_o,	initial absolute temperature;
T_s,	sphere temperature;
T_{so},	initial sphere temperature;
Ts,	saturation temperature;
Tw,	sphere surface temperature;
W,	dimensionless quantity, $W = 1+(m-1)p_{v0}$;
W_m,	value of W at pressure's first maximum.

Greek symbols

α,	dimensionless parameter, $\alpha = [p_{v0}k_v(Tw - T_\infty)/\rho_{v0}\Lambda][\rho R/P_\infty\delta_0^3]^{1/2}$;
α_s,	sphere thermal diffusivity, $\alpha_s = k_s/\rho_s c_s$;
β,	dimensionless parameter, $\beta^2 = (p_{v0}k\rho/k_v\rho_{v0})[B/(T_w - T_\infty)][BC/\Lambda]$;
γ,	dimensionless parameter, $\gamma = (\delta_0/R)kB/[k_v(T_w - T_\infty)]$;
Δ,	thermal disturbance's penetration depth into sphere;
δ,	film thickness;
δ_0,	initial film thickness;
Λ,	heat of vaporization;
ρ,	liquid density;
ρ_s,	sphere density;
ρ_{v0},	initial vapor density;
τ,	dimensionless time, $\tau = At$;
τ_m,	value of τ at pressure's first maximum.

INTRODUCTION

THE CONSEQUENCES of suddenly bringing a very hot substance into contact with a large pool of cool liquid have been explored in recent times by a number of investigators. A major motivation of many of these studies is the fact that under certain conditions vapor-explosions of violent nature occur.

There are a number of physical mechanisms which could be involved in a vapor-explosion, acting either singly or in combination. Reid [1] in a survey article, primarily devoted to the case where the hot substance is molten, emphasizes the possibility that the nearby liquid in the pool might first become superheated with a later explosive formation of vapor. Fauske [2, 3] has proposed that liquid droplets entrained in the molten hot substance can first become superheated and later explosively achieve the vapor state. Epstein [4] discusses the possibility that the violent release of dissolved gas within the molten metal upon cooling may be responsible for the molten metal's fragmentation and sudden increase in heat transfer, explaining why vapor explosions often do not occur when the molten substance is heated in an inert atmosphere. It has also been proposed by Hsiao *et al.* [5] and

Cronenberg *et al.* [6] that the thermal stresses caused by contraction of a solid crust forming on the molten material could pressurize the molten core and cause fragmentation, leading to sudden increases in heat transfer. Zyszkowski [7] extended the thermal stress mechanism postulate by presuming that thermal stresses were greatly increased in importance if the crust forming on the molten material is an oxide, explaining in a manner different from Epstein the lack of occurance of vapor explosions when the molten material is heated in an inert atmosphere. The magnitude and duration of the heat-transfer rate between the hot substance and the cool liquid are particularly important to the ability of the proposed dissolved-gas-release and thermal-stress phenomena to account for fragmentation of the molten substance and vapor-explosions, but they have not heretofore been evaluated in a general way despite prior recognition of their importance by Cho and Wright [8]. Experiments concerning the interaction of molten droplets with a liquid pool are cited in the foregoing references.

Experiments to study the behavior of the thin vapor film separating a hot solid from the cool liquid in which it is immersed have been reported by Stevens and Witte [9, 10] and Walford [11]. They observed that the enveloping vapor films sometimes suddenly became unstable in a manner suggestive of a vapor-explosion; Witte *et al.* [12] suggested that the forces arising from such an effect could induce molten drop fragmentation. A related analytical study of the growth of an initially thin vapor film enveloping a hot solid sphere immersed in a liquid pool was performed by Kazimi, Todreas, Lanning, and Rohsenow [13]. They executed a detailed numerical solution of the describing equations for the case of a subcooled liquid and accounted for heat conduction in the liquid and the solid, vapor and liquid compressibility, and the presence of some noncondensable gas in the vapor film which was of nonzero initial thickness. Among their findings was that pressure excursions of as much as 10 times the ambient pressure and appreciable decreases in the hot solid's surface temperature could result in some realistic cases. However, the purely numerical nature of their investigation makes their results difficult to apply to cases different from those they studied. Rooney [14] considered the same problem with the simplifying assumptions that the liquid is saturated, the sphere is of constant temperature, and heat conduction in the liquid is negligible, and found that the vapor film's pressure excursions could be predicted by general and simple analytical relations. Because of Rooney's simplifying assumptions, his predictions for the magnitude of pressure excursions must be regarded as upper bounds. Kibbee [15] extended Rooney's analysis to include the effect of heat conduction in the saturated liquid and showed that the pressure excursions were still predictable by general and simple analytical relations.

In the present work, the analysis of Kibbee is extended to include the effect of a subcooled liquid pool upon the pressure in the vapor film enveloping a hot solid sphere suddenly exposed to a cool liquid pool. Predictive analytical relations will be derived for the film pressure's maximum excursion and related quantities which generalize the more detailed numerical study of Kazimi *et al.* The results of the present work are of interest not only because of their possible application to the vapor-explosion problem, which more often occurs with a subcooled than a saturated liquid pool, but more generally because they also reveal the effects of a compressible vapor in a transient change-of-phase problem and allow rapid estimation of the conditions under which this compressibility is either important or negligible.

PROBLEM FORMULATION

A very hot solid sphere of constant temperature is suddenly exposed to a large pool of stagnant and subcooled liquid. Initially, a thin film of vapor and noncondensable gas envelopes the sphere, preventing direct contact between the sphere and the liquid. Because the conductive heat flow from the sphere into the liquid interface across the intervening film exceeds the conductive heat flow from the liquid interface into the pool, additional vapor is formed. The great disparity between the vapor and liquid densities requires that the film's pressure increase to create additional space for the vapor by a radial displacement of the liquid.

As the liquid is radially displaced the vaporization rate is diminished below its initial value because the heat flow rate from the sphere diminishes, primarily because of the increased film thickness, while the heat conduction from the liquid interface into the pool increases due to the dependence of saturation temperature upon pressure. Since the sphere temperature greatly exceeds the temperature of the vapor–liquid interface the temperature variation of the sphere's surface, which would be small due to the small elapsed time, would not appreciably affect the rate of heat flow from the sphere. The larger the subcooling of the liquid pool, the smaller will be the first pressure excursion and the slower will be the increase in film thickness since heat conduction into the liquid from the interface will then be greater for a specified total pressure in the film. Because of its inertia, the liquid's radial displacement eventually becomes too large so that the total pressure in the film decreases below the ambient value. The liquid then accelerates toward the sphere and the film thickness, total pressure, and heat flow rate from the sphere are oscillatory.

In the following analysis a spherical film of vapor and noncondensable gas, always thin enough to have a linear temperature distribution and negligible curvature, is assumed to initially separate the sphere from the stagnant and incompressible liquid with both gases assumed to be perfect and uniformly distributed. Gravitational body forces are neglected. The liquid and the vapor are assumed to be in equilibrium at their interface with the saturation temperature varying logarithmically with the instantaneous vapor partial pressure (the possibility of a layer of superheated liquid near the interface is not allowed by this assumption). Finally, it is assumed that all quantities depend only upon time and radial position.

As shown by Kibbee, the one-dimensional equations of motion and continuity with application of the conservation of mass at the interface lead to

$$\begin{aligned} &\mathrm{d}^2(\delta/\delta_0)/\mathrm{d}\tau^2-(\rho_{v0}/\rho)\mathrm{d}^2m/\mathrm{d}\tau^2 \\ &\quad+[\delta_0/(R+\delta)]\,[\mathrm{d}(\delta/\delta_0)/\mathrm{d}\tau \\ &\quad-(\rho_{v0}/\rho)\mathrm{d}m/\mathrm{d}\tau]\,[3\mathrm{d}(\delta/\delta_0)/\mathrm{d}\tau \\ &\qquad\qquad+(\rho_{v0}/\rho)\mathrm{d}m/\mathrm{d}\tau]/2 = p-1. \quad (1) \end{aligned}$$

An integral solution of the liquid's one-dimensional energy equation, using a quadratic temperature profile, with application of the conservation of energy at the interface in conjunction with a heat flow rate from the sphere given by

$$q_v = k_v(T_w-T_s)/\delta \quad (2)$$

leads to

$$\begin{aligned} p_{v0}\mathrm{d}m/\mathrm{d}\tau = {}&\alpha[1-\gamma\ln(p_v/p_{v0})]/(\delta/\delta_0) \\ &-\beta(4\alpha/3)^{1/2}\mathrm{d}I/\mathrm{d}\tau \quad (3) \end{aligned}$$

where

$$I^2 = \int_0^\tau \mathit{ln}^2(p_v/p_{v0})\mathrm{d}\tau. \quad (4)$$

The initial conditions imposed are that the film is initially of non-zero thickness

$$\delta(\tau=0)/\delta_0 = 1 \quad (5)$$

so that

$$m(\tau=0) = 1. \quad (6)$$

The total film pressure is initially equal to the ambient pressure

$$p(\tau=0) = 1 \quad (7)$$

and the liquid is initially stagnant

$$\mathrm{d}[\delta(\tau=0)/\delta_0]/\mathrm{d}\tau = (\rho_{v0}/\rho)\mathrm{d}m(\tau=0)/\mathrm{d}\tau. \quad (8)$$

When the ratio of instantaneous to initial film thickness (δ/δ_0) is related to the ratio of instantaneous total film pressure to ambient pressure (p) and the ratio of instantaneous vapor mass per unit area to initial vapor mass per unit area (m), equations (1)–(8) allow the temporal variation of dimensionless film pressure, mass, and thickness to be determined. Assuming the film's vapor and noncondensable gas to be perfect and uniformly distributed gives

$$\begin{aligned} \delta/\delta_0 = [(M_gR_g+MR_v)T/P]/[(M_gR_g \\ +M_0R_v)T_0/P_\infty]. \end{aligned}$$

Recognizing that the average temperature in the film varies but little allows the above relation to be recast as

$$\begin{aligned} \delta/\delta_0 = [(M_gR_g/M_0R_v+M/M_0)(P_\infty/P)]/ \\ [M_gR_g/M_0R_v+1]. \quad (9) \end{aligned}$$

Since the noncondensable gas and the vapor are uniformly distributed in the film, the perfect gas equation of state also gives

$$T_0M_gR_g/P_{g0} = T_0M_0R_v/P_{v0}$$

or

$$M_gR_g/M_0R_v = P_{g0}/P_{v0}. \quad (10)$$

Inasmuch as the initial total pressure is the sum of the initial partial pressure of the vapor and noncondensable gas

$$P_\infty = P_{g0}+P_{v0} \quad (11)$$

Equations (10) and (11) substituted into equation (9) then give the sought after relation in terms of dimensionless quantities as

$$\delta/\delta_0 = [1+(m-1)p_{v0}]/p. \quad (12)$$

Substituting equation (12) into the initial condition of equation (8) allows the effect of the ratio of vapor to liquid density to be explored since the result is

$$\mathrm{d}p(\tau=0)/\mathrm{d}\tau = \alpha(1-\rho_{v0}/\rho p_{v0}). \quad (13)$$

Insamuch as the initial vapor density is always much less than the liquid density, $\rho_{v0} \lll \rho$, equation (13) is accurately represented by

$$\mathrm{d}p(\tau=0)/\mathrm{d}\tau = \alpha. \quad (14)$$

It is also necessary to relate the ratio of instantaneous to initial vapor pressure in the film to the total pressure and vapor mass in the film for use in equation (4). Starting with the fact that the total pressure is the sum of the vapor and noncondensable gas partial pressures

$$P = P_v+P_g$$

and noting additionally that

$$P_g = P_{g0}\delta/\delta_0$$

with δ/δ_0 given by equation (12) results in

$$p_v/p_{v0} = mp/[1+(m-1)p_{v0}]. \quad (15)$$

Because $\rho_{v0} \lll \rho$ and $\delta_0 \lll R$, after substituting equation (12) into equations (1) and (14) and equation (15) into equation (4), the final equations to be solved are

$$p_{v0}\mathrm{d}m/\mathrm{d}\tau = \alpha p/[1+(m-1)p_{v0}]-\beta(4\alpha/3)^{1/2}\mathrm{d}I/\mathrm{d}\tau \quad (16)$$

$$\mathrm{d}^2\{[1+(m-1)p_{v0}]/p\}/\mathrm{d}\tau^2 = p-1 \quad (17)$$

where

$$I^2 = \int_0^\tau \ln^2\{mp/[1+(m-1)p_{v0}]\}\,\mathrm{d}\tau \quad (18)$$

subject to the initial conditions of

$$m(\tau = 0) = 1 = p(\tau = 0) \tag{18a}$$

and

$$\mathrm{d}(\{1+[m(\tau = 0)-1]p_{v0}\}/p(\tau = 0))/\mathrm{d}\tau = 0. \tag{19}$$

SOLUTIONS

As discussed by Kibbee for the saturated liquid case, it is expected that the magnitude of the film pressure's first excursion would increase with increasing values of the parameter α and would decrease with increasing values of the parameter β. It is expected that decreasing values of p_{v0}, an additional parameter corresponding to increasing subcooling of the liquid pool, would decrease the magnitude of the film's first pressure excursion for two reasons. First, for low values of p_{v0} the film is initially mostly filled with the noncondensable gas which acts as a cushion. Second, for low values of p_{v0} the vapor's instantaneous partial pressure has experienced a substantial increase for any appreciable increase in the film's total pressure; the saturation temperature at the interface has correspondingly undergone a substantial increase, resulting in increased heat conduction into the liquid from the interface and a reduced rate of film pressure increase.

Numerical method

Equations (16)–(19) were solved numerically on a Honeywell 66/60 digital computer by use of the MIMIC [16] program. The step size used in the numerical integration was automatically adjusted to maintain a prescribed accuracy.

The emphasis of the present study is on the events up to and including the time at which film pressure achieves its first maximum, for which purpose the integral solution of the liquid's energy equation is satisfactory since the interface temperature then increases monotonically. Later events are of some interest, however. The changing direction of liquid heat conduction at the interface was generally accounted for in an approximate way by using

$$\mathrm{d}I/\mathrm{d}\tau \simeq |\ln(p_v/p_{v0})|\ln(p_v/p_{v0})/2I$$

in equation (18). Although the possible inaccuracy of the integral method when interface temperature varies nonmonotonically is recognized [17], adequate accuracy is expected since heat conduction in the liquid is expected to be of less importance after the first pressure excursion.

Small excursions

For small pressure excursions such as would be expected when the parameter α is small, a simple approximate solution for the film's pressure excursion and its oscillation frequency can be found following Kibbee's procedure. First, let $W = 1+(m-1)p_{v0}$. The equations (16)–(19) become

$$\mathrm{d}W/\mathrm{d}\tau = \alpha p/W - \beta(4\alpha/3)^{1/2}(\mathrm{d}I/\mathrm{d}W)(\mathrm{d}W/\mathrm{d}\tau) \tag{20}$$

$$\mathrm{d}^2W/\mathrm{d}\tau^2 = p-1 \tag{21}$$

$$W(\tau = 0) = 1 = p(\tau = 0) \tag{22}$$

$$\mathrm{d}[W(\tau = 0)/p(\tau = 0]/\mathrm{d}\tau = 0. \tag{23}$$

Following Kibbee's procedure which was suggested by the results of the numerical solutions described earlier, an average value of $\mathrm{d}I/\mathrm{d}W$ is assumed so that equation (20) becomes

$$\mathrm{d}W/\mathrm{d}\tau = Fp/W \tag{24}$$

where

$$F = \alpha/[1+\beta(4\alpha/3)^{1/2}(\mathrm{d}I/\mathrm{d}W|_{\mathrm{ave}})].$$

Rooney's results can now be used directly (α in his expressions is replaced by F) as

$$\ln(p) = FW^{-1/4}\sin[2(W^{3/2}-1)/3F] \tag{25}$$

$$p_m = 1+F \tag{26}$$

$$W_m = (1+3\pi F/4)^{2/3} \tag{27}$$

$$\tau_m = \pi/2 \tag{28}$$

and, if there is need of them, his expressions for frequency can be similarly taken over.

From its definition $\mathrm{d}I/\mathrm{d}W|_{\mathrm{ave}}$ can now be evaluated since

$$I^2 = \int_1^W \ln^2[p(W-1+p_{v0})/(Wp_{v0})](\mathrm{d}\tau/\mathrm{d}W)\,\mathrm{d}W.$$

For $p \simeq 1$, equation (24) shows that $\mathrm{d}\tau/\mathrm{d}W = W/F$. In addition W does not depart greatly from its initial value of unity. These two approximations substituted into the above relation for I^2 give

$$FI^2 \simeq \int_1^W [\ln(p)+(W-1)(1-p_{v0})/(Wp_{v0})]^2\,W\mathrm{d}W.$$

Into this relation equation (25) is substituted giving, with $x = 2(W^{3/2}-1)/3F$,

$$I^2/F^2 \simeq \int_0^x [\sin(x)+x(1-p_{v0})/p_{v0}]^2\,\mathrm{d}x$$

from which it is found that

$$I^2/F^2 \simeq [2x-\sin(2x)]/4 + 2(1-p_{v0})\sin(x)-x\cos(x)]/p_{v0}+(1-p_{v0})^2x^3/3p_{v0}.$$

Now,

$$\mathrm{d}I/\mathrm{d}W|_{\mathrm{ave}} = I(W_m)/(W_m-1)$$

which, in conjunction with equation (27) and the above relation for I^2, gives

$$\mathrm{d}I/\mathrm{d}W|_{\mathrm{ave}} \simeq [1+8(1-p_{v0})/\pi p_{v0}+(1-p_{v0})^2\pi^2/6p_{v0}^2]^{1/2}/\pi^{1/2}.$$

Thus, from equation (26) it is finally seen that for small pressure escursions

$$p_m \simeq 1+\alpha/\{1+\beta(4\alpha/3\pi)^{1/2}[1+8(1-p_{v0})/\pi p_{v0} + (1-p_{v0})^2\pi^2/6p_{v0}^2]^{1/2}\}. \tag{29}$$

The form of this result shows that the three parameters which govern the magnitude of the first (and largest) pressure excursion are α and β (both of which are substantially independent of initial vapor pressure) and p_{v0}. Equation (29) indicates that the magnitude of the first pressure excursion decreases as the initial vapor pressure decreases (liquid subcooling increases).

Large excursions

An approximate solution for the magnitude of the film pressure's first excursion can be found for the large excursions that would be expected when α is large by a procedure similar to Kibbee's. For large excursions equations (21) and (24) are closely approximated by

$$\mathrm{d}^2(W/p)/\mathrm{d}\tau^2 = p \qquad (30)$$

and

$$\mathrm{d}W/\mathrm{d}\tau = Fp/W \qquad (24)$$

where F is as defined before, except for the need to reevaluate $\mathrm{d}I/\mathrm{d}W|_{\mathrm{ave}}$, and the initial conditions of equations (22) and (23) are unchanged.

Rooney gives the accompanying solutions as

$$p = We^{-W^3/6F^2} \qquad (31)$$

$$p_m = (2F^2/\mathrm{e})^{1/3} \qquad (32)$$

$$W_m = (2F^2)^{1/3} \qquad (33)$$

$$\tau_m = (2/F)^{1/3} \qquad (34)$$

$$\delta_m/\delta_0 = \mathrm{e}^{1/3}. \qquad (35)$$

Proceeding to evaluation of $\mathrm{d}I/\mathrm{d}W|_{\mathrm{ave}}$, the definition of I^2 together with equation (24) gives

$$FI^2 = \int_1^W \ln^2[(W-1+p_{v0})p/(Wp_{v0})]\,(W/p)\,\mathrm{d}W.$$

With the use of equation (31) and letting

$$K = \int_0^1 \mathrm{e}^{-x^3/3}\,\mathrm{d}x = 1/1.082,$$

$$C = \int_0^1 \mathrm{e}^{x^3/3}\,\mathrm{d}x = 1.092,$$

and

$$y = K(W-1+p_{v0})/p_{v0}$$

the above relation becomes

$$KFI^2/Cp_{v0} = \int_K^y \ln^2(y)\,\mathrm{d}y$$

from which it is found that

$$I^2 = (CW_m/F)([1-(1-p_{v0})/W_m]\{\ln^2[K(W_m-1+p_{v0})/(\mathrm{e}p_{v0})]+1\} - (p_{v0}/W_m)[\ln^2(K/\mathrm{e})+1]).$$

Since $\mathrm{d}I/\mathrm{d}W|_{\mathrm{ave}} = I(W_m)/(W_m-1)$, expressing F and W_m in terms of p_m from equations (32) and (33)

$$\mathrm{d}I/\mathrm{d}W|_{\mathrm{ave}} = (C^{1/2}2^{1/4}/\mathrm{e}^{5/12}p_m{}^{5/4}) \times\{[1-(1-p_{v0})/(\mathrm{e}^{1/3}p_m)] \times(\ln^2\{K[p_m-(1-p_{v0})/\mathrm{e}^{1/3}]/(\mathrm{e}^{2/3}p_{v0})\}+1) -[p_{v0}/(\mathrm{e}^{1/3}p_m)][\ln^2(K/\mathrm{e})+1]\}^{1/2}.$$

This relation substituted into the definition of F which is in turn substituted into equation (32) gives the final implicit relation for p_m as

$$p_m^{3/2}+0.946p_m^{1/4}\alpha^{1/2}\beta\{[1-(1-p_{v0})/(1.396p_m)] \times(\ln^2\{[p_m-0.717(1-p_{v0})]/(2.11p_{v0})\}+1) -1.55p_{v0}/p_m\}^{1/2} = (2/\mathrm{e})^{1/2}\alpha. \qquad (36)$$

Inspection of equation (36) reveals that in the limit as α becomes very large, since p_m increases as α increases and since $\ln(p_m)/p_m^{5/4}$ then tends toward zero,

$$p_m = (2/3)^{1/3}\alpha^{2/3}$$

which is the result Rooney obtained in the absence of heat conduction for a saturated liquid.

Surface temperature change of sphere

A possible small decrease in the sphere's surface temperature and increase in the interface's temperature, changes which affect the values of both α and β, can be accounted for by an iterative technique. First p_m and the time required to achieve it are determined based on a first estimate of α and β. The average interface temperature and the value of β are then corrected; the sphere's surface temperature is corrected by the relations [18] for a semi-infinite solid initially at uniform temperature and subjected to the constant heat flux calculated from knowledge of the initial film thickness and temperature difference. New values of α and β are then computed from which new values of p_m and related quantities are determined. Iterations are executed in this manner until the results no longer show appreciable change.

Simple general expressions for the temperature change of the sphere's surface and the depth to which a thermal disturbance penetrates into the sphere are of interest, not only for the iterations discussed above, but also because of their applicability to the dissolved-gas-release and thermal-stress hypotheses for vapor-explosions. An integral solution to the one-dimensional heat conduction equation for a semi-infinite solid initially at uniform temperature and subjected to a time dependent heat flux is obtained in the following way. The appropriate integral form of the heat conduction equation is

$$\mathrm{d}\Big[\int_0^\Delta (T_s-T_{s0})\mathrm{d}z\Big]/\mathrm{d}t = -q_v/\rho_s C_s.$$

With a quadratic temperature profile in the semi-infinite solid of

$$T_s-T_{s0} = -(q_v\Delta/2k_s)(1-z/\Delta)^2$$

it is found that

$$T_w - T_{s0} = -(q_{v0}/A^{1/2})(3/2k_s\rho_s C_s)^{1/2} \left[(q_v/q_{v0})\int_0^\tau (q_v/q_{v0})\,\mathrm{d}\tau\right]^{1/2} \quad (37)$$

and

$$\Delta = (6\alpha_s/A)^{1/2}\left[(q_{v0}/q_v)\int_0^\tau (q_v/q_{v0})\mathrm{d}\tau\right]^{1/2}. \quad (38)$$

The integral method used in obtaining these results is accurate during the time in which film pressure increases since then the film thickness, and the heat flow rate from the sphere, do not vary greatly. At later times, when film pressure decreases from its first excursion due to an increasing film thickness, these results of the simple integral method can be substantially in error; however, because the heat flow rate from the sphere is also greatly diminished then, the loss in accuracy is of less concern than might at first be thought.

The change in sphere surface temperature and the thermal disturbance's penetration depth can be related to the previously determined maximum pressure excursion. In general, equation (2) gives

$$q_v/q_{v0} = \delta_0/\delta$$

which, by virtue of equations (12) and (31) for large pressure excursions, becomes

$$q_v/q_{v0} = \mathrm{e}^{-W^3/6F2}. \quad (39)$$

With

$$i = (q_v/q_{v0})\int_0^\tau (q_v/q_{v0})\mathrm{d}\tau$$

and using equations (24) and (39) to conveniently express $\mathrm{d}\tau/\mathrm{d}W$ and q_v/q_{v0}, one has

$$i = (W-1)\mathrm{e}^{-W^3/6F^2}$$

which, in consideration of equation (32), has a maximum value of

$$i = (2/ep_m)^{1/2}$$

at $W = W_m$, or at $p = p_m$. This result substituted into equation (37) yields the sphere's maximum surface temperature change for large pressure excursions as

$$T_w - T_{s0} = -[3/(2e)^{1/2}]^{1/2}[q_{v0}/(k_s\rho_s c_s)^{1/2}]/(Ap_m^{1/2})^{1/2}. \quad (40)$$

Proceeding in like fashion to evaluate the integral in equation (38) gives the thermal disturbance's penetration depth at the time of greatest change in sphere surface temperature for large pressure excursions as

$$\Delta = [3(2^{1/2})\mathrm{e}^{1/6}]^{1/2}\,\alpha_s^{1/2}/(Ap_m^{1/2})^{1/2}. \quad (41)$$

For small pressure excursions ($p \simeq 1$) equation (24) indicates that $W \simeq (1+2F\tau)^{1/2}$. With $p \simeq 1$, equation (12) shows that $\delta/\delta_0 \simeq W$ so that

$$q_v/q_{v0} \simeq (1+2F\tau)^{-1/2}$$

and

$$i = [1 - 1/(1+2F\tau)^{1/2}]/F$$

which when substituted into equation (37) gives the change in sphere surface temperature as asymptotically approaching a maximum value of, since $F = p_m^{-1}$,

$$T_W - T_{s0} = -[3/2]^{1/2}[q_{v0}/(k_s\rho_s C_s)^{1/2}]/[A(p_m - 1)]^{1/2}.$$

Similarly, the corresponding penetration depth of the thermal disturbance follows from equation (38) as asymptotically approaching

$$\Delta = 6^{1/2}\alpha_s^{1/2}(1+2F\tau)^{1/2}/(AF)^{1/2}.$$

It can be seen from the expression for i that the maximum change in sphere surface temperature is nearly achieved at $\tau = 4/F = 4/(p_m - 1)$, at which time

$$\Delta = 6\alpha_s^{1/2}/[A(p_m - 1)]^{1/2}. \quad (42)$$

DISCUSSION

The results of the numerical calculations are displayed in Fig. 1 along with the approximate solutions. Since the parameters α and β are only weakly dependent on the initial partial pressure of the vapor, the effect of liquid subcooling is nearly isolated in the parameter p_{v0}. It can be seen that liquid subcooling substantially reduces the magnitude of pressure excursions.

To illustrate the application of these results, consider a tin sphere of 0.3 cm radius initially at a uniform temperature of 500°C suddenly put a stagnant pool of water whose ambient pressure is $1.01325 \times 10^5\,\mathrm{N/m^2}$. The thickness of the initial vapor film whose existence is postulated, perhaps due to the sphere's immersion from a gaseous environment, is taken to be 10^{-5} cm. Under these conditions $\beta = 13$, $\alpha = 3200$, and $A = 0.61 \times 10^6\,s^{-1}$. From Fig. 1 as well as equation (36), the magnitude of the first film pressure excursion and its time of occurance are: $11.6 \times 10^5\,\mathrm{N/m^2}$ at 0.55×10^{-6} s for $p_{v0} = 1$ (OC subcooled liquid), $9.44 \times 10^5\,\mathrm{N/m^2}$ at 0.78×10^{-6} s for $p_{v0} = 0.467$ (20°C subcooled liquid), $5.24 \times 10^5\,\mathrm{N/m^2}$ at 1.0×10^{-6} s for $p_{v0} = 0.122$ (50°C subcooled liquid), and $3.22 \times 10^5\,\mathrm{N/m^2}$ at 1.4×10^{-6} s for $p_{v0} = 0.0325$ (80°C subcooled liquid). Figure 2 displays the film pressure from the numerical solution for 20°C and 50°C subcooling along with the results of Kazimi's calculations; although there are greater differences between the results of the two studies than could be hoped for, primarily because saturation temperature varies a bit more strongly than logarithmically with vapor pressure, the agreement is still believed to be substantial in view of the generality of the present study and to lend credence to the present study. The decrease in surface temperature and the thermal disturbance's penetration depth for the tin sphere at the time of maximum pressure are found from

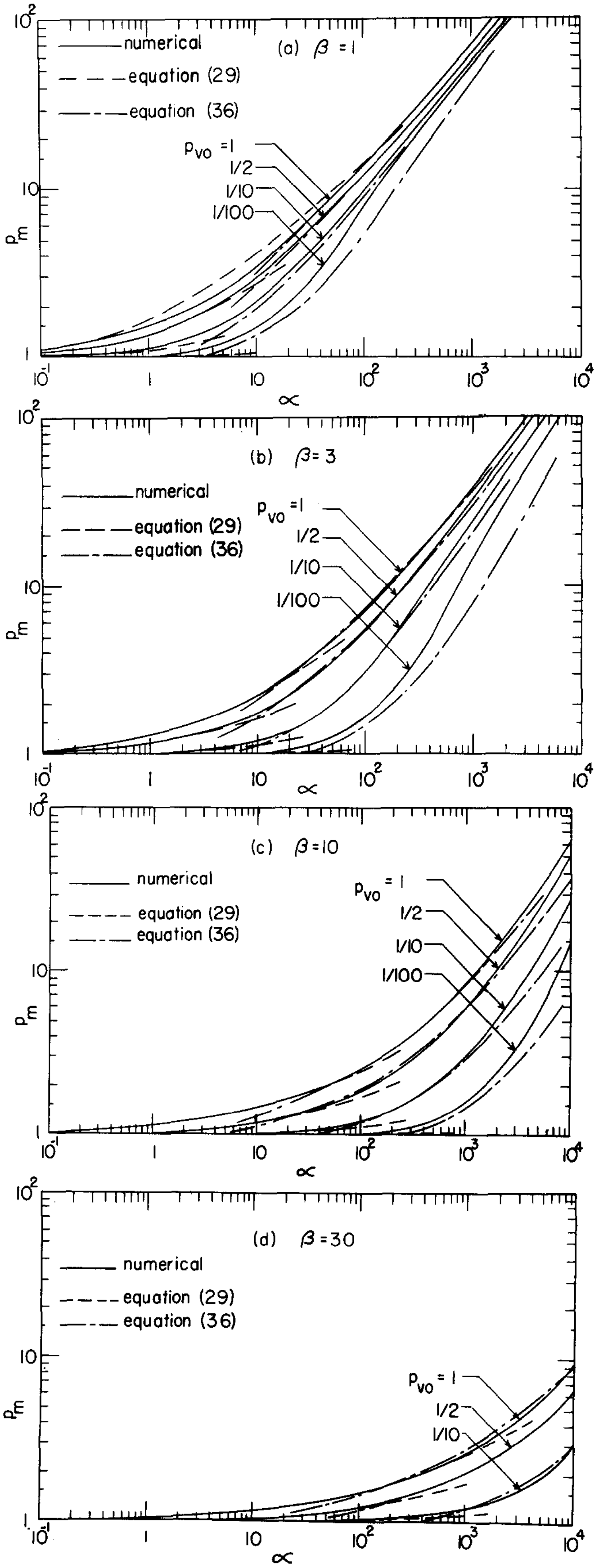

FIG. 1. Maximum dimensionless pressure vs α with p_{v0} as a parameter for: (a) $\beta = 1$; (b) $\beta = 3$; (c) $\beta = 10$; (d) $\beta = 30$.

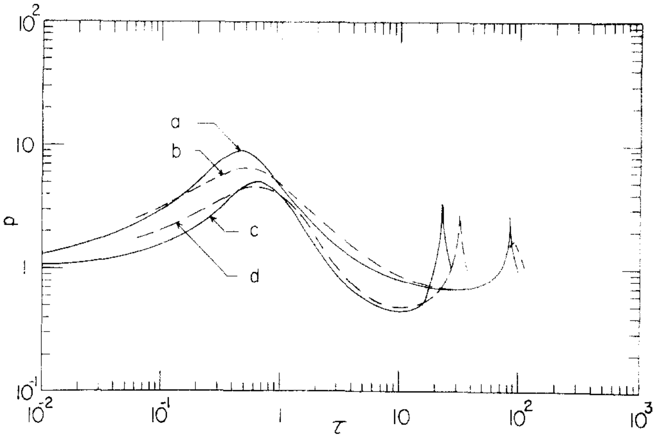

FIG. 2. Dimensionless total film pressure vs dimensionless time with $\alpha = 3200$ and $\beta = 13$ for: (a) present study at 20°C water subcooling; (b) Kazimi *et al.* [13] at 20°C water subcooling; (c) present study at 50°C water subcooling; and (d) Kazimi *et al.* [13] at 50°C water subcooling.

equations (40) and (41) to be small, about 15°C and 10^{-3} cm, respectively. Had the liquid initially been in direct contact with the hot solid, an immediate and larger decrease in sphere surface temperature of about 60°C is predicted by the classical formula [18]

$$T_W - T_{s0} = -(T_{s0} - T_\infty)/[1 + (k_s\rho_sC_s/k\rho C)^{1/2}].$$

The difference between the surface temperature change predicted by the present study and the direct liquid contact formula increases with increasing temperature difference between the sphere and the liquid.

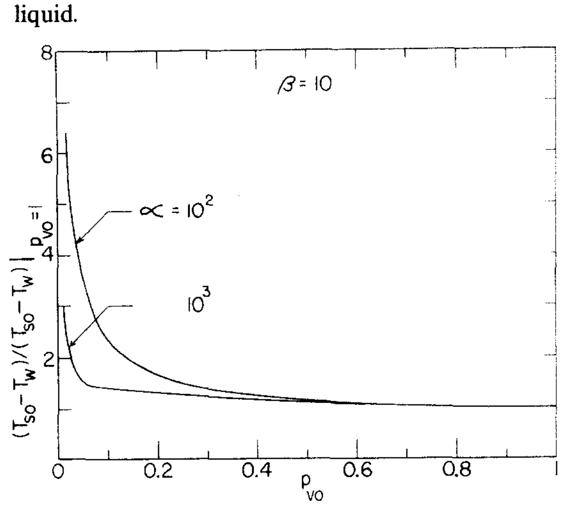

FIG. 3. Dimensionless drop of sphere's surface temperature vs dimensionless initial vapor pressure.

The dependence of surface temperature decrease on the initial partial vapor pressure in the film was determined from the numerical solutions and is shown in Fig. 3 for several values of α and a representative value of $\beta = 10$. It is seen there that the sphere's surface temperature drop rapidly increases as p_{v0} decreases below a threshold value, suggesting that the likelihood of vapor-explosions increases rapidly when liquid subcooling increases above a threshold value if hypotheses dependent on quenching rate such as dissolved-gas-release or thermal-stress are correct. The numerical solutions show that as subcooling increases above a threshold value the heat flow rate from the sphere remains near its initial high value longer so that the thermal disturbance's penetration depth into the sphere increases along with the increased surface temperature drop. The increased subcooling reduces the vaporization rate, in other words, and allows the film to remain thin for greater periods of time. Figure 3 also shows that the threshold subcooling of the liquid pool becomes greater as α increases.

In applying the results of the present study, one should be aware that the thickness of the gaseous film assumed to initially separate the solid from the liquid is difficult to specify. Further, the time required for film pressure to achieve its first maximum is short enough (of the order of 10^{-6} s in many cases) that liquid compressibility is of some importance as shown by Kazimi; non-Fourier effects on heat conduction [19] are found to be unimportant even at these small times, however. It is also noted that the vaporization rates for the thin initial films assumed in this study do not exceed the maximum predicted from kinetic theory [20], although they do approach it in some cases. If the sphere's presence in the liquid pool is conceived to occur by dropping, it is unlikely that complete immersion would occur before film pressure achieved its first maximum.

Some proposed vapor explosion mechanisms, such as the thermal-stress hypothesis (which has been challenged by Ladisch [21]), are most likely to be active when the sphere is initially molten and is deformable. Buchanan [22], for example, postulates the penetration of a molten sphere by a liquid jet from the pool after collapse of a vapor film. Buxton and Nelson [23] support the idea that vapor film collapse following development of an instability in the vapor–liquid interface triggers a vapor explosion, particularly in the case of a deformable molten sphere. The assumptions of the present study that the sphere is undeformable and that the geometry is spherical with no vapor escape should be viewed with caution as idealizations that, while allowing representative pressure excursions and heat fluxes of importance to the dissolved-gas-release and thermal stress hypotheses to be determined, may be physically realized only imperfectly.

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REFERENCES

1. R. Reid, Superheated liquids, *Am. Scient.* **64**(2), 146–156 (1976).
2. H. K. Fauske, On the mechanism of uranium dioxide–sodium explosive interactions, *Nucl. Sci. Engng* **51**, 95–101 (1973).
3. S. G. Bankoff and H. K. Fauske, On the mechanisms for vapor explosions, *Nucl. Sci. Engng* **54**, 481–482 (1974).
4. M. Epstein, Thermal fragmentation—a gas release phenomenon, *Nucl. Sci. Engng* **55**, 462–467 (1974).
5. K. H. Hsiao, J. E. Cox, P. G. Hedgcoxe and L. C. Witte, Pressurization of a solidifying sphere, *J. Appl. Mech.* **39E**(1), 71–77 (1972).
6. A. W. Cronenberg, T. C. Chawla and H. K. Fauske, Thermal stress mechanism for the fragmentation of molten UO_2 upon contact with sodium coolant, *Nucl. Engng Des.* **30**(3), 434–443 (1974).
7. W. Zyszkowski, Study of the thermal explosion phenomena in molten copper–water system, *Int. J. Heat Mass Transfer* **19**(8), 849–868 (1976).
8. D. H. Cho and R. W. Wright, A rate limited model of molten fuel-coolant interactions, *Trans. Am. Nucl. Soc.* **13**(2), 659–660 (1970).
9. J. W. Stevens and L. C. Witte, Transient film and transition boiling around spheres, *Int. J. Heat Mass Transfer* **14**, 443–450 (1971).
10. J. W. Stevens and L. C. Witte, Transient vapor film behavior during quenching, *Trans. Am. Nucl. Soc.* **15**, 418–420 (1972).
11. F. J. Walford, Transient heat transfer from a hot nickel sphere moving through water, *Int. J. Heat Mass Transfer* **12**(12), 1621–1625 (1969).
12. L. C. Witte, J. E. Cox and J. E. Bouvier, The vapor explosion, *J. Metals* **22**(2), 39–44 (1970).
13. M. S. Kazimi, N. E. Todreas, D. D. Lanning and W. M. Robsenow, A theoretical study of the dynamic growth of a vapor film around a hot sphere in a coolant, *Fifth International Heat Transfer Conference*, Sept. 1974, Tokyo, Japan (Heat Transfer 1974).
14. M. Rooney and L. Burmeister, Pressure excursions in transient film boiling from a sphere, *Int. J. Heat Mass Transfer* **18**, 671–675 (1975).
15. S. E. Kibbee, J. A. Orozco and L. Burmeister, Influence of liquid heat conduction on maximum pressure during transient film boiling from a sphere to a saturated liquid, *Int. J. Heat Mass Transfer* **20**, 1069–1075 (1977).
16. F. J. Sansom and H. E. Peterson, MIMIC programming manual, Technical Report SEG-TR-67-31, Systems Engineering Group, Aeronautical Systems Division, Air Force Systems Command, Wright–Patterson Air Force Base, Ohio (July 1967).
17. T. R. Goodman and N. Ullah, Heat balance integral methods for pulse-like heat inputs, AFOSR 2261 (1962).
18. H. S. Carslaw and J. L. Jaeger, *Heat Conduction in Solids*, 2nd Edn, pp. 75 and 88. Oxford University Press, Oxford (1959).
19. M. J. Maurer and H. A. Thomson, Non-Fourier effects at high heat flux, *J. Heat Transfer* **95**(2), 284 (1973).
20. J. R. Maa, Evaporation coefficient of liquids, *IEC Fundamentals* **6**(4), 504 (1967).
21. R. Ladisch, Comment on fragmentation of UO_2 by thermal stress and pressurization, *Nucl. Engng Des.* **43**, 327–328 (1977).
22. D. J. Buchanan, A model for fuel-coolant interactions, *J. Phys., D:Appl. Phys.* **7**, 1441–1457 (1974).
23. L. D. Buxton and L. S. Nelson, Impulse-initiated gas release—a possible trigger for vapor explosions, *Trans. Am. Nucl. Soc.* **26**, 398–399.

EVOLUTION DE LA PRESSION DANS L'EBULLITION EN FILM TRANSITOIRE POUR UNE SPHERE DANS UN LIQUIDE SOUS-REFROIDI

Résumé—Les variations maximales de pression qui se produisent dans le film de vapeur formé autour d'une sphère solide très chaude, à température presque constante, et brusquement exposée à un liquide sous-refroidi et stagnant sont évaluées analytiquement à partir des grandeurs caractéristiques. On présente sous forme graphique et par une formule approachée les prédictions généralisées des fréquences d'oscillation de la pression du film et la variation maximale. La chute de température à la surface de la sphère est estimée de façon approchée et elle devient appréciable quand le sous-refroidissement du liquide dépasse une valeur de seuil.

DRUCKSCHWANKUNGEN BEIM INSTATIONÄREN FILMSIEDEN AN EINER KUGEL IN EINER UNTERKÜHLTEN FLÜSSIGKEIT

Zusammenfassung—Eine sehr heiße Kugel aus einem Feststoff von nahezu konstanter Temperatur wird plötzlich in einen mit unterkühlter Flüssigkeit gefüllten Behälter getaucht. Die mazimalen Druckspitzen in dem die Kugel umgebenden Dampffilm werden analytisch berechnet. In grafischer Form und durch Näherungsgleichungen werden verallgemeinerte Aussagen über die Schwankungsfrequenz und die maximalen Amplituden des Filmdrucks gemacht. Der Temperaturabfall an der Oberfläche der Kugel wird abgeschätzt; er wird bedeutend, wenn die Unterkühlung der Flüssigkeit einen bestimmten Grenzwert überschreitet.

ИЗМЕНЕНИЕ ДАВЛЕНИЯ ПРИ НЕСТАЦИОНАРНОМ ПЛЕНОЧНОМ КИПЕНИИ НА ПОВЕРХНОСТИ СФЕРЫ, ПОГРУЖЕННОЙ В НЕДОГРЕТУЮ ЖИДКОСТЬ

Аннотация — Аналитически исследуется пленочное кипение и максимальное изменение давления в пленке пара на поверхности очень горячей твердой сферы почти постоянной температуры, внезапно погружаемой в объём неподвижной и недогретой жидкости. На основании расчётов представлены графики и апроксимирующие уравнения для частоты колебания и максимального изменения давления в пленке. Дана приближенная оценка перепада температуры на поверхности сферы и показано его существенное увеличение в случае, когда недогрев жидкости превышает пороговое значение.